48660 Dynamics and Control

Project 1 - Part 1: Modelling of a practical dynamic system (i.e. coupled-tank system)

The Coupled Tanks System (Figure 1) emulates an engineering scenario where it is critical to maintain a desired fluid level. The coupled tanks system can have single or multiple inputs and output(s). Students are asked to characterise the behaviour of the system (find the transfer function of the plant). The rigs were designed to allow students to acquire data from a physical dynamic system to develop a simplified model of the underlying dynamics.

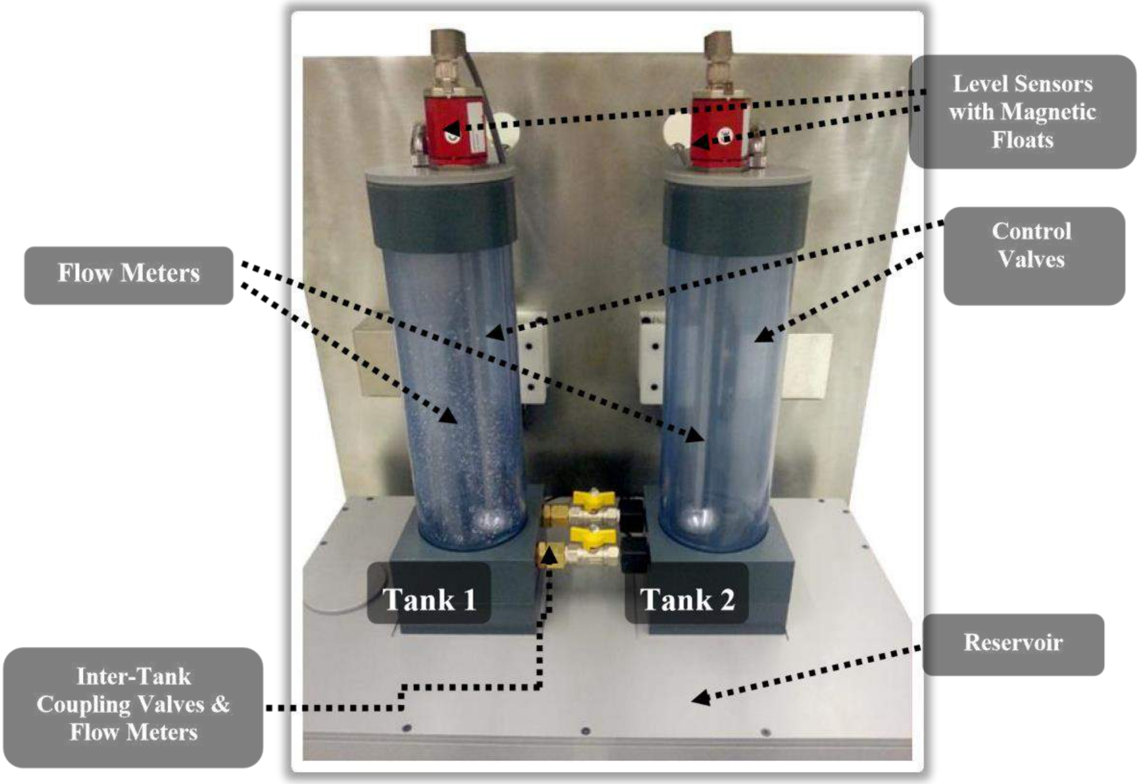


Figure 1 - Coupled Tanks Rig, Generation II

Once the model is developed in Project 1 – Part 1, students will be asked (in Project 1 – Parts 2 & 3) to design controllers for the dynamic system and analyse the performance of the controllers in maintaining the water level in Tank 2.

You are required to read the document "Coupled_Tanks_Generation II_UserGuide_V2-3.pdf" for detailed information about the coupled tank system.

Students are required to produce a report detailing the following four tasks:

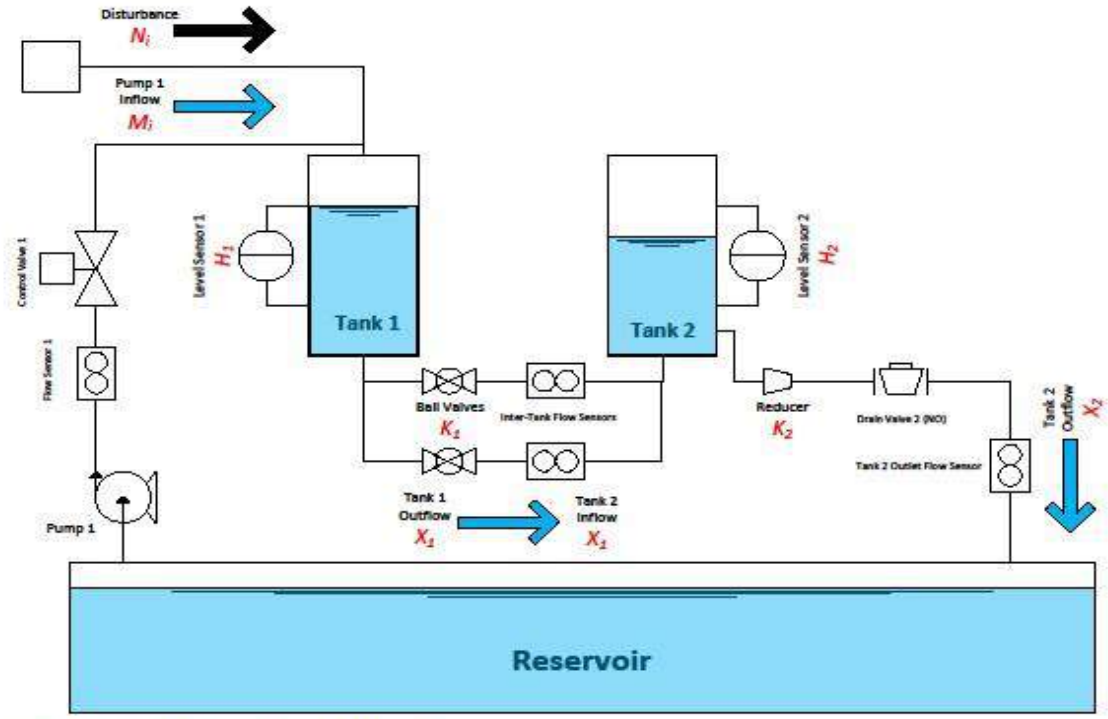
Task 1: Determine the Inputs and Outputs of 3 Key System Components

Using the calibration data ("Coupled_Tanks_Calibration_Data_2016-08-04.pdf"), produce the component block diagrams for Sensor 2, Valve 1 and the Coupled Tanks. Diagrams for each key system component should be similar in format to that presented in Figure 2, with inputs and outputs fully defined.

Figure 2 - Component Block Diagram

Task 2: Develop the Transfer Functions of 3 Key System Components

Using the calibration data ("Coupled_Tanks_Calibration_Data_2016-08-04.pdf"), develop the transfer functions of Sensor 2 and Valve 1.



Tank Dynamics & Properties Identification:

M_i = Tank 1 Flow Rate In

N_i = Disturbance to Tank 1 Flow Rate In [±]

X_1 = Tank 1 Flow Rate Out = Tank 2 Flow Rate In

X_2 = Tank 2 Flow Rate Out

K_1 = Flow/Head Constant (Flow Resistance Out Of Tank 1)

K_2 = Flow/Head Constant (Flow Resistance Out Of Tank 2)

A_1 = Tank 1 Cross Sectional Area

A_2 = Tank 2 Cross Sectional Area

Tank Dynamics Equations:

Tank 1 Net Flow IN	$= M(t) + N(t)$
Tank 1 Net Flow OUT	$= X_1(t) = K_1 * H_1(t)$
Tank 2 Net Flow IN	$= X_1(t) = K_1 * H_1(t)$
Tank 2 Net Flow OUT	$= X_2(t) = K_2 * H_2(t)$

Tank 1 Equation:

$$M(t) + N(t) - X_1(t) = A_1[dH_1/dt]$$

Tank 2 Equation:

$$X_1(t) - X_2(t) = A_2[dH_2/dt]$$

Figure 3 - Coupled Tanks Dynamics and Properties

Develop the differential equations and the transfer function of the coupled tanks system (using the equations for Tank 1 and Tank 2 in Figure 3). At steady-state mass flow rate in is equal to mass flow rate out.

Combining the two differential equations (*Tank 1 Equation* and *Tank 2 Equation* shown in Figure 3) and then taking Laplace transform (or taking Laplace transform of the two equations and then combining the two linear equations), the transfer function of the coupled tanks takes the form of:

$$H_2(s) = \frac{M(s)+N(s)}{k_2T_1T_2s^2 + k_2(T_1+T_2)s + k_2} = \frac{M(s)+N(s)}{Js^2 + as + k_2}$$

Important: *Students are required to derive this equation and show the derivation steps!*

Where:

$$k_2 = K_2$$

$$T_1 = \frac{A_1}{K_1}$$

$$T_2 = \frac{A_2}{K_2}$$

$$J = k_2T_1T_2$$

$$a = k_2(T_1 + T_2)$$

T_1 and T_2 are time constants that are related to the cross-sectional area of the tanks, the operating levels in the tanks and the difference in levels in the tanks.

To determine the values of J, a and k_2, follow the steps detailed below which uses an empirical approach.

Step 1: Measure the Open-loop Response of the Coupled Tanks Using the Remote Lab

Students are required to measure the open-loop response of the coupled water tanks from the coupled tank test rigs in the UTS remote labs. To do this, students must:

1. Login to the remote coupled-tanks (refer to log in instructions on UTS Online)
2. Select “Manual” on the interface
3. Check the “Valve %” is 0; and make sure the water level is at its minimum (normally less than 10mm), otherwise wait until the water level is decreased to the minimum
4. Click “Logging” and select format

5. Set the “Valve %” from 0 to 20% (or from 0 to 15% if the tanks overflow. But you need to wait until the tanks are empty)
6. Wait until the water level in Tank 2 (H_2) reaches steady-state[1] (about 700-800 seconds)
7. Set the “Valve %” to 0
8. Download the session file(s)
9. Use the data from the session file to generate a graph of the open loop response, plotting (***in the same graph***, similar to Figure 4):
    a. Water level (mm) vs time (s)
    b. Flow rate (L/min) vs time (s)

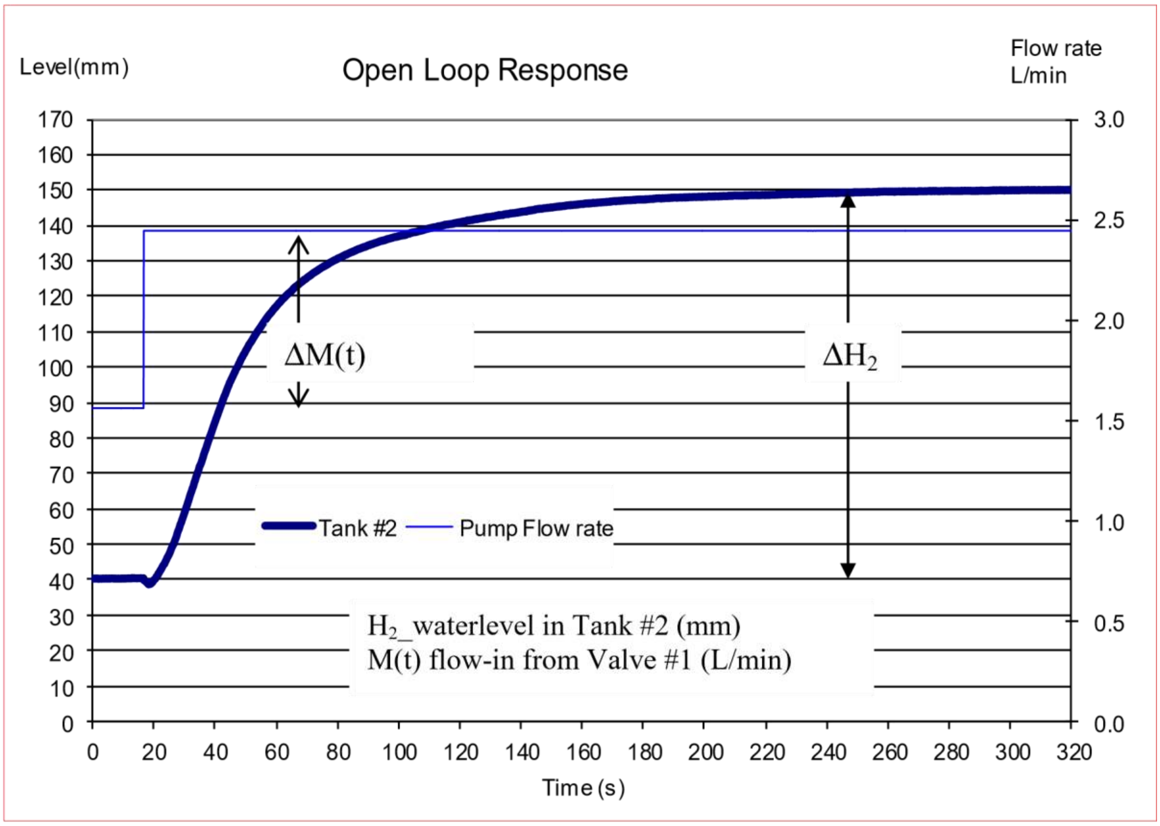


Figure 4 - An Example of an Open Loop Response

Figure 4 shows an **EXAMPLE** of an open loop response, which illustrates how the water level in Tank 2 changes when the flow into the system changes.

[1] Steady-state is defined as “an unvarying condition in a physical process”. In the context of the coupled tanks system, this refers to the change in water level in Tank 2 (H_2) over each time step being very close to zero (i.e. reaching a plateau).

Step 2: Determine the Values of J, a, and k_2

Parameters J, a and k_2 can be calculated based on the measured open-loop response of the level in Tank 2. Any one of the three methods described below may be used to determine J, a and k_2.

In Project 1 – Parts 2 & 3, the model developed in Part 1 will be used to design a controller for the coupled tank system. It should be noted that the parameters obtained through these methods are empirical and therefore have limited accuracy. The key limiting value is $\sum T_{\varepsilon 2}$ as it imposes the limit on what can be achieved. If $\sum T_{\varepsilon 2}$ is *underestimated* then the resulting controller may be too fast for the system's dynamics and the closed loop will be severely underdamped or even unstable.

Method 1

k_2 can be defined as*:* $k_2 = \frac{\Delta M}{\Delta H_2}$

For J and a we need estimates of the time constants T_1 and T_2 that comprise the second order dynamics. The two time constants are related to the operating levels in the tanks, the difference in levels in the tanks and are directly proportional to the cross-sectional area of the tanks.

We define that: $t_{63\%} = T_1 + T_2 + \sum T_{\varepsilon 2}$. Assuming $\sum T_{\varepsilon 2}$ be of the order of 1 to 3 seconds.

Next, estimate T_1 which is the larger of the two time constants. One way of doing this is to measure the time for the level response to be 86% complete ($t_{86\%}$). Then:

$$T_1 \approx t_{86\%} - t_{63\%}$$

$$T_2 \approx t_{63\%} - T_1 - \sum T_{\varepsilon 2}$$

Note: $t_{86\%}$ and $t_{63\%}$ are the times corresponding to the times when the water level of Tank 2 reaches 86% and 63% of the steady-state height, respectively (not 86% and 63% of the total experiment run time). Additionally, the start time of the experiment may not be zero, it should be the time when the "Valve %" is set to 20% (or 15%). It is common for there to be an initial height of water in tank 2 which can't be drained. When calculating the steady state value for the height, this initial water height needs to be taken into consideration. This steady state value is calculated as: $h_{2,ss} = h_{2,f} - h_{2,i}$.

Method 2

Another estimate [1] is to draw the line of steepest slope through the response curve and locate the time where this line cuts the base line (Figure 5). Call this L. Then:

$$T_1 \approx t_{63\%} - L$$

$$T_2 \approx L - \sum T_{\varepsilon 2}$$

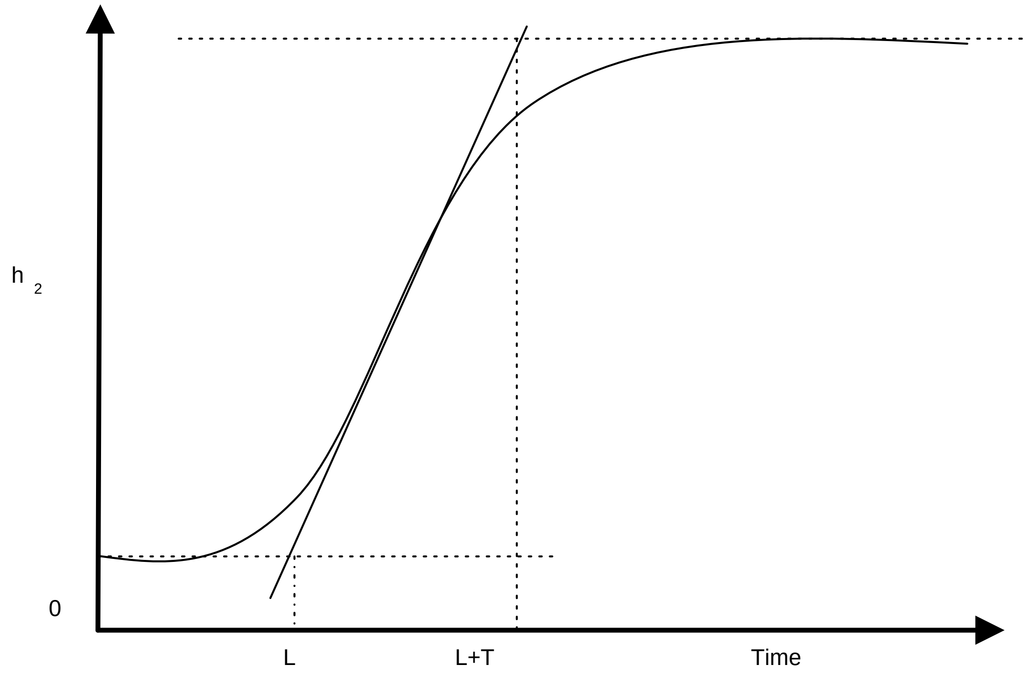


Figure 5 - Method 2 Example

Method 3

A third method [1][2] uses the same graph as shown in Figure 5 and assumes:

$$T_1 = T$$

$$T_2 = L$$

And make:

$$J = k_2 T_1 T_2$$

$$a = k_2 (T_1 + T_2)$$

Task 3: Develop the Closed-Loop Control System Block Diagram

Using the calculated transfer functions, signal flow and units from Task 1 and Task 2, develop the closed-loop control system block diagram for the control of the water level in Tank 2. The input to the system should be a desired water level for Tank 2 (mm) and the output should be the actual water level in Tank 2 (mm). Note that closed-loop is comprised of the following components: A controller (which will be developed in Project 1 – Parts 2 & 3), Valve 1, Sensor 2 and the Coupled Tanks.

Task 4: Discussion and Reflection

Provide an insightful, clear, relevant but brief discussion and reflection on the tasks performed in this report. Draw some conclusions about why modelling such a system might be useful in real life engineering practice.

Additional Notes

- Students must include units for all quantities measured or derived. Use *seconds* as the time unit for the dynamics. The unit for (s) in Laplace domain is (1/*seconds*).
- Students should consult the marking guide provided on UTSOnline.
- The main body of the reports must be limited to 8 pages or less (i.e. not including title page, table of contents etc.).
- Scanned hand written reports are acceptable, so long as they are neat and easily readable. Any hand written work should be written on blank, unruled paper and scanned in only. Digital photographs of hand written work will attract deductions for the report presentation. Graphs and figures should be made using software and annotated appropriately (with legends, titles, captions, axis labels).
- Submission is via UTS Online under the Assignments tab; follow the instructions provided.

References

[1] K. Stillman, Control Engineering I, UTS Lecture notes, 1996
[2] H.T. Nguyen, Analogue and Digital Control, UTS Lecture notes, 1998